**Derivation of the formulae for calculating a four component cement raw mix.**

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 To solve a four component raw mix calculation requires four simultaneous equations of the form;

a\*W + b\*X + c\*Y + d\*Z = e

f \*W + g\*X + h\*Y + i\*Z = j

k\*W + l\*X + m\*Y + n\*Z = o

p\*W + q\*X + r\*Y + s\*Z = t

 Where the letters W, X, Y and Z represent the unknown percentages of each of the four raw materials, and the letters *a, b ,c ,d ,e, f, g, h, I, j, k, l, m, n, o, p, q, r, s* and *t*  are numerical coefficients.

Since the sum of the four component percentages in any raw mix is always 100%, Therefore we can write the first equation as;- W + X + Y + Z = 100

 Therefore, in the equations above;- a =1, b=1, c=1, d=1 and e =100. However, we still need to provide three more equations. These equations come from the three quality control parameters LSF, SR and AR which are critical inputs into the raw mix calculation. Each of these three parameters can be calculated using the chemical analysis of the four components and the percentage of each in the raw mix.

 So, if we let the chemical analysis of each component be…

|  |  |  |  |
| --- | --- | --- | --- |
| **Component W** | **Component X** | **Component Y** | **Component Z** |
| SiO2 = S1 | SiO2 = S2 | SiO2 = S3 | SiO2 = S4 |
| Al2O3 = A1 | Al2O3 = A2 | Al2O3 = A3 | Al2O3 = A4 |
| Fe2O3 = F1 | Fe2O3 = F2 | Fe2O3 = F3 | Fe2O3 = F4 |
| CaO = C1 | CaO = C2 | CaO = C3 | CaO = C4 |

 …then the AR of the raw mix can be calculated as;-

AR = (W\*A1 + X\*A2 + Y\*A3 + Z\*A4) / (W\*F1 + X\*F2 + Y\*F3 + Z\*F4)

∴

AR \* (W\*F1 + X\*F2 + Y\*F3 + Z\*F4) = (W\*A1 + X\*A2 + Y\*A3 + Z\*A4)

∴

AR\*W\*F1 + AR\*X\*F2 + AR\*Y\*F3 + AR\*Z\*F4 = (W\*A1 + X\*A2 + Y\*A3 + Z\*A4

∴

(AR\*F1 -A1) \* w + (AR\*F2 – A2) \* x + (AR\*F3 – A3) \* Y + (AR\*F4 – A4)\*Z = 0

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 We now have the second simultaneous equation in the form;- f\*W + g\*X + h\*Y + i\*Z = j

 Where;-

f = (AR\*F1 - A1)

g = (AR\*F2 – A2)

h = (AR\*F3 – A3)

i = (AR\*F4 – A4)

j = 0

 The third equation comes from evaluating the SR, which can be calculated as;-

SR = (W\*S1 + X\*S2 + Y\*S3 + Z\*S4) / (W\*A1 + X\*A2 + Y\*A3 + Z\*A4 + W\*F1 + X\*F2 + Y\*F3 + Z\*F4)

∴

SR \* (W\*A1 + X\*A2 + Y\*A3 + Z\*A4 + W\*F1 + X\*F2 + Y\*F3 + Z\*F4) = (W\*S1 + X\*S2 + Y\*S3 + Z\*S4)

∴

(SR\*A1 + SR\*F1 – S1)\*W + (SR\*A2 + SR\*F2 – S2)\*X + (SR\*A3 + SR\*F3 – S3)\*Y + (SR\*A4 + SR\*F4 – S4)\*Z = 0

 This is the third simultaneous equation and is in the form;- k\*W + l\*X + m\*Y + n\*Z = o

 Where;-

k = (SR\*A1 + SR\*F1 – S1)

l = (SR\*A2 + SR\*F2 – S2)

m = (SR\*A3 + SR\*F3 – S3)

n = (SR\*A4 + SR\*F4 – S4)

o = 0

 The fourth and last simultaneous equation is derived from the LSF, which can be calculated as;-

LSF = (100 \* (W\*C1 + X\*C2 + Y\*C3 + Z\*C4)) / (2.8\*(W\*S1 + X\*S2 + Y\*S3 + Z\*S4) + 1.18\*(W\*A1 + X\*A2 + Y\*A3 + Z\*A4) +0.65\*(W\*F1 + X\*F2 + Y\*F3 + Z\*F4))

∴

LSF \* (2.8\*(W\*S1 + X\*S2 + Y\*S3 + Z\*S4) + 1.18\*(W\*A1 + X\*A2 + Y\*A3 + Z\*A4) +0.65\*(W\*F1 + X\*F2 + Y\*F3 + Z\*F4)) = 100 \* (W\*C1 + X\*C2 + Y\*C3 + Z\*C4)

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∴

LSF\* (2.8\*W\*S1 + 2.8\*X\*S2 +2.8\*Y\*S3 +2.8\*Z\*S4 + 1.18\*W\*A1 + 1.18\*X\*A2 + 1.18\*Y\*A3 + 1.18\*Z\*A4 +0.65\*W\*F1 + 0.65\*X\*F2 + 0.65\*Y\*F3 + 0.65\*Z\*F4) = 100\*W\*C1 +100\*X\*C2 + 100\*Y\*C3 + 100\*Z\*C4

∴

(LSF\*2.8\*S1 + LSF\*1.18\*A1 +LSF\*0.65\*F1 – 100\*C1) \* W +

(LSF\*2.8\*S2 + LSF\*1.18\*A2 +LSF\*0.65\*F2 – 100\*C2) \* X +

(LSF\*2.8\*S3 + LSF\*1.18\*A3 +LSF\*0.65\*F3 – 100\*C3) \* Y +

(LSF\*2.8\*S4 + LSF\*1.18\*A4 +LSF\*0.65\*F4 – 100\*C4) \* Z +

=0

 This is the fourth equation we need, and is in the form;- p\*W + q\*X + r\*Y + s\*Z = t

 Where;-

p = (LSF\*2.8\*S1 + LSF\*1.18\*A1 +LSF\*0.65\*F1 – 100\*C1)

q = LSF\*2.8\*S2 + LSF\*1.18\*A2 +LSF\*0.65\*F2 – 100\*C2)

r = (LSF\*2.8\*S3 + LSF\*1.18\*A3 +LSF\*0.65\*F3 – 100\*C3)

s = (LSF\*2.8\*S4 + LSF\*1.18\*A4 +LSF\*0.65\*F4 – 100\*C4)

t = 0

 The four simultaneous equations…

a\*W + b\*X + c\*Y + d\*Z = e

f \*W + g\*X + h\*Y + i\*Z = j

k\*W + l\*X + m\*Y + n\*Z = o

p\*W + q\*X + r\*Y + s\*Z = t

 … can be expressed in matrix form as;-

$\left( \begin{matrix}a&b&c&d\\f&g&h&i\\k&l&m&n\\p&q&r&s\end{matrix} \right)$ $\left(\begin{matrix} W\\X\\Y\\Z\end{matrix}\right)$ = $\left(\begin{matrix} e \\j\\o\\t\end{matrix}\right)$

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 By using this matrix form of our four simultaneous equations we can solve them by using Cramer’s rule. (See <http://math.tutorvista.com/algebra/cramers-rule.html>)

 Cramer’s rule can be used with square matrices (ie 2x2, 3x3, 4x4… etc) to individually calculate the unknowns in simultaneous equations by using matrix determinants;-

 The determinant of the coefficient matrix in our case is expressed as;-

D = $\left|\begin{matrix}a&b&c&d\\f&g&h&i\\k&l&m&n\\p&q&r&s\end{matrix}\right|$

 …and the determinants of the four unknowns W, X, Y & Z are expressed as ;-

Dw = $\left|\begin{matrix}e&b&c&d\\j&g&h&i\\o&l&m&n\\t&q&r&s\end{matrix}\right|$

Dx = $\left|\begin{matrix}a&e&c&d\\f&j&h&i\\k&o&m&n\\p&t&r&s\end{matrix}\right|$

Dy = $\left|\begin{matrix}a&b&e&d\\f&g&j&i\\k&l&o&n\\p&q&t&s\end{matrix}\right|$

D z = $\left|\begin{matrix}a&b&c&e\\f&g&h&j\\k&l&m&o\\p&q&r&t\end{matrix}\right|$

 Using these five determinants, Cramer’s rule allows us to calculate the values of the unknowns in our four simultaneous equations via the following formulae;-

W = Dw/ D

X = Dx / D

Y = Dy / D

Z = Dz / D

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 The determinants D, Dw, Dx, Dy & Dz are evaluated by cross multiplication which in this case gives the following formulae for each of the determinants ;-

D = (a\*g\*m\*s)+(a\*h\*n\*q)+(a\*i\*l\*r)-(a\*i\*m\*q)-(a\*g\*n\*r)-(a\*h\*l\*s)-(f\*b\*m\*s)-(f\*c\*n\*q)-(f\*d\*l\*r)+(f\*d\*m\*q)+(f\*b\*n\*r)+(f\*c\*l\*s)+(k\*b\*h\*s)+(k\*c\*i\*q)+(k\*d\*g\*r)-(k\*d\*h\*q)-(k\*b\*i\*r)-(k\*c\*g\*s)-(p\*b\*h\*n)-(p\*c\*i\*l)-(p\*d\*g\*m)+(p\*d\*h\*l)+(p\*b\*i\*m)+(p\*c\*g\*n)

Dw= (e\*g\*m\*s)+(e\*h\*n\*q)+(e\*i\*l\*r)-(e\*i\*m\*q)-(e\*g\*n\*r)-(e\*h\*l\*s)-(j\*b\*m\*s)-(j\*c\*n\*q)-(j\*d\*l\*r)+(j\*d\*m\*q)+(j\*b\*n\*r)+(j\*c\*l\*s)+(o\*b\*h\*s)+(o\*c\*i\*q)+(o\*d\*g\*r)-(o\*d\*h\*q)-(o\*b\*i\*r)-(o\*c\*g\*s)-(t\*b\*h\*n)-(t\*c\*i\*l)-(t\*d\*g\*m)+(t\*d\*h\*l)+(t\*b\*i\*m)+(t\*c\*g\*n)

Dx = (a\*j\*m\*s)+(a\*h\*n\*t)+(a\*i\*o\*r)-(a\*i\*m\*t)-(a\*j\*n\*r)-(a\*h\*o\*s)-(f\*e\*m\*s)-(f\*c\*n\*t)-(f\*d\*o\*r)+(f\*d\*m\*t)+(f\*e\*n\*r)+(f\*c\*o\*s)+(k\*e\*h\*s)+(k\*c\*i\*t)+(k\*d\*j\*r)-(k\*d\*h\*t)-(k\*e\*i\*r)-(k\*c\*j\*s)-(p\*e\*h\*n)-(p\*c\*i\*o)-(p\*d\*j\*m)+(p\*d\*h\*o)+(p\*e\*i\*m)+(p\*c\*j\*n)

Dy = (a\*g\*o\*s)+(a\*j\*n\*q)+(a\*i\*l\*t)-(a\*i\*o\*q)-(a\*g\*n\*t)-(a\*j\*l\*s)-(f\*b\*o\*s)-(f\*e\*n\*q)-(f\*d\*l\*t)+(f\*d\*o\*q)+(f\*b\*n\*t)+(f\*e\*l\*s)+(k\*b\*j\*s)+(k\*e\*i\*q)+(k\*d\*g\*t)-(k\*d\*j\*q)-(k\*b\*i\*t)-(k\*e\*g\*s)-(p\*b\*j\*n)-(p\*e\*i\*l)-(p\*d\*g\*o)+(p\*d\*j\*l)+(p\*b\*i\*o)+(p\*e\*g\*n)

Dz = (a\*g\*m\*t)+(a\*h\*o\*q)+(a\*j\*l\*r)-(a\*j\*m\*q)-(a\*g\*o\*r)-(a\*h\*l\*t)-(f\*b\*m\*t)-(f\*c\*o\*q)-(f\*e\*l\*r)+(f\*e\*m\*q)+(f\*b\*o\*r)+(f\*c\*l\*t)+(k\*b\*h\*t)+(k\*c\*j\*q)+(k\*e\*g\*r)-(k\*e\*h\*q)-(k\*b\*j\*r)-(k\*c\*g\*t)-(p\*b\*h\*o)-(p\*c\*j\*l)-(p\*e\*g\*m)+(p\*e\*h\*l)+(p\*b\*j\*m)+(p\*c\*g\*o)

 Where;

a = 1

b = 1

c = 1

d = 1

e = 100

f = (AR\*F1 - A1)

g = (AR\*F2 – A2)

h = (AR\*F3 – A3)

i = (AR\*F4 – A4)

j = 0

k = (SR\*A1 + SR\*F1 – S1)

l = (SR\*A2 + SR\*F2 – S2)

m = (SR\*A3 + SR\*F3 – S3)

n = (SR\*A4 + SR\*F4 – S4)

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o = 0

p = (LSF\*2.8\*S1 + LSF\*1.18\*A1 +LSF\*0.65\*F1 – 100\*C1)

q = LSF\*2.8\*S2 + LSF\*1.18\*A2 +LSF\*0.65\*F2 – 100\*C2)

r = (LSF\*2.8\*S3 + LSF\*1.18\*A3 +LSF\*0.65\*F3 – 100\*C3)

s = (LSF\*2.8\*S4 + LSF\*1.18\*A4 +LSF\*0.65\*F4 – 100\*C4)

t = 0

 Before substituting these coefficients into the determinant formulae we can simplify them because, in our case, the coefficients ***j , o*** and ***t*** are all equal to zero. This means that any term in the determinants which contains any of these coefficients as a multiplier can be removed, because that term will always evaluate to zero. This significantly simplifies the four determinants of the unknowns, and they become;-

Dw = (e\*g\*m\*s)+(e\*h\*n\*q)+(e\*i\*l\*r)-(e\*i\*m\*q)-(e\*g\*n\*r)-(e\*h\*l\*s)

Dx = -(f\*e\*m\*s)+(f\*e\*n\*r)+(k\*e\*h\*s) -(k\*e\*i\*r)-(p\*e\*h\*n)+(p\*e\*i\*m)

Dy = -(f\*e\*n\*q)+(f\*e\*l\*s)+(k\*e\*i\*q)-(k\*e\*g\*s)-(p\*e\*i\*l)+(p\*e\*g\*n)

Dz =-(f\*e\*l\*r)+(f\*e\*m\*q)+(k\*e\*g\*r)-(k\*e\*h\*q)-(p\*e\*g\*m)+(p\*e\*h\*l)

 Also , since a = b = c = d = 1, these letters can be removed as multipliers from the formula for D i.e.;-

D = (g\*m\*s)+(h\*n\*q)+(i\*l\*r)-(i\*m\*q)-(g\*n\*r)-(h\*l\*s)-(f\*m\*s)-(f\*n\*q)-(f\*l\*r)+(f\*m\*q)+(f\*n\*r)+(f\*l\*s)+(k\*h\*s)+(k\*i\*q)+(k\*g\*r)-(k\*h\*q)-(k\*i\*r)-(k\*g\*s)-(p\*h\*n)-(p\*i\*l)-(p\*g\*m)+(p\*h\*l)+(p\*i\*m)+(p\*g\*n)

 Now, by substituting the coefficient values into the simplified equations for D, Dw, Dx, Dy and Dz, the percentages of the mix components W, X, Y, & Z can be calculated using the formulae;-

W = Dw/ D

X = Dx / D

Y = Dy / D

Z = Dz / D

 Finally, to simplify the calculation and allow automatic recalculation of raw mix percentages when changing any of the raw material chemistry variables or the LSF, SR and AR targets these calculation above and the coefficients (*a* thru s) can be incorporated into a simple Excel spreadsheet. **(**See Appendix 1 on the next page for an example)

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 **Appendix 1**



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